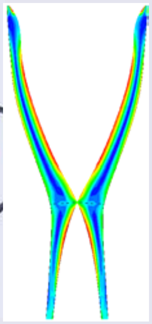
John Rasmussen's topics

- Computer-aided Design
- Design optimization
  - CAOS
  - ODESSY
- Biomechanics
  - The AnyBody project
  - AnyBody Technology
- Common denominator: Optimization.



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Introduction to Optimization

- Mathematical interpretation
  - Implicit and explicit problems
  - Terminology
- Problem types
- Algorithm types

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Optimization is:

- About minimization of functions under certain constraints.
- A method to solve mathematical problems.
- Solution of
  - Equations
  - Systems of equations
  - Differential equations
  - Systems of differential equations
- Can handle under-determinate problems.
- Can handle over-determinate problems.

A very wide class of problems!

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Optimization as an analysis tool

We usually consider optimization as something one step further than analysis. But optimization can also be analysis. Many natural phenomena are based on optimality, and analyzing them entails using this property actively.

- Mechanical systems come to rest at a state of minimum energy
- Drops of water take on shapes determined by minimum surface tension energy in concert with the surrounding air pressure
- Biological systems strive to minimize energy consumption to maximize the chance of survival
- Solution of equations can be formulated as the minimization of solution error.

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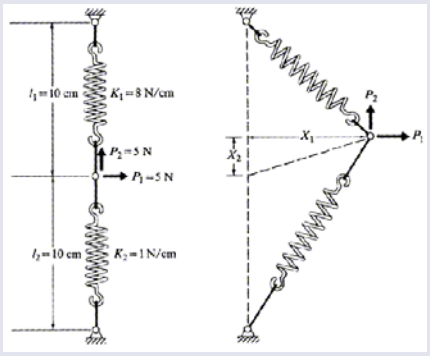
Mechanical systems in equilibrium

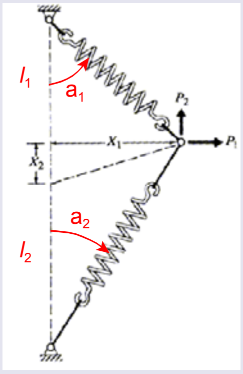


The system comes to rest:

- where the spring forces are in equilibrium with P1 and P2
- where the total potential energy is minimum.

Either principle will give us the solution.

Traditional solution



Two nonlinear equations with two unknowns.

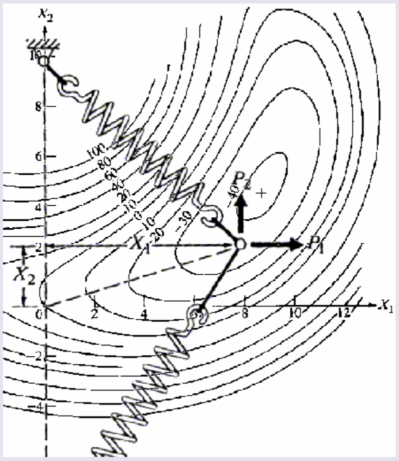
$$\frac{K1\left(\sqrt{X1^2+(l1-X2)^2}-l1\right)(l1-X2)}{\sqrt{X1^2+l1^2-2\,l1\,X2+X2^2}}+\frac{K2\left(\sqrt{X1^2+(l2+X2)^2}-l2\right)(l2+X2)}{\sqrt{X1^2+l2^2+2\,l2\,X2+X2^2}}=P1$$

$$\frac{K2\left(\sqrt{X1^2+(l2+X2)^2}-l2\right)X1}{\sqrt{X1^2+l2^2+2\,l2\,X2+X2^2}}-\frac{K1\left(\sqrt{X1^2+(l1-X2)^2}-l1\right)X1}{\sqrt{X1^2+l1^2-2\,l1\,X2+X2^2}}=P2$$

We insert the numbers, solve the equations, and obtain
X1 = 8.24;   X2 = 4.76.

This is a rather complicated way of doing it.

Minimum potential energy



Mechanical systems come to rest where their potential energy is minimal.

The potential energy can be expressed as:

$$E = \frac{1}{2}K_1\left[\left(\sqrt{X_1^2 + (l_1 - X_2)^2} - l_1\right)^2\right]$$
$$+\frac{1}{2}K_2\left[\left(\sqrt{X_1^2 + (l_2 + X_2)^2} - l_2\right)^2\right]$$
$$-P_1X_1 - P_2X_2$$

This is obviously much simpler.

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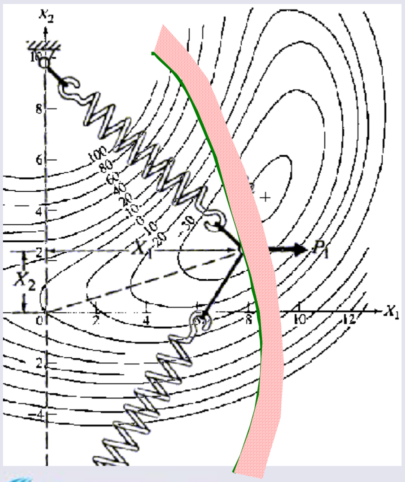
Contact condition



Contact conditions complicate a solution with equilibrium equations enormously. Optimization is much simpler.

The objective function is the same. Now we just have to add a constraint on X_1 and X_2 corresponding to the contact wall.

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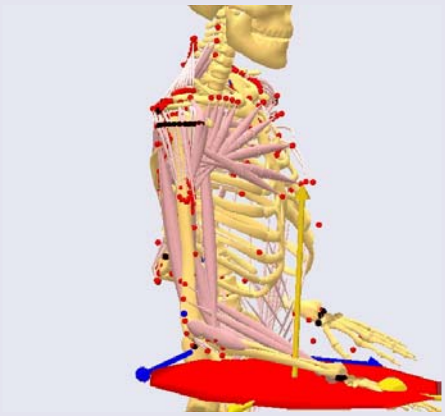
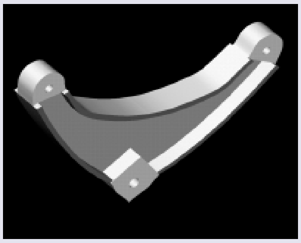
Optimization and bioechanica analysis

- Biological systems act according to optimality criteria: The survival of the fittest.
- This can be used to create simulations of the human musculoskeletal system.

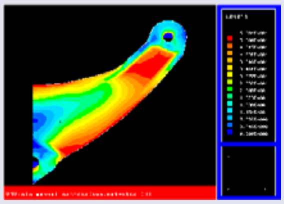


Explicit and implicit

$\rightarrow \boxed{V = hD^2/4} \rightarrow$ 0.5 ℓ



$\rightarrow$ FE analysis $\rightarrow$



Unconstrained optimization

Minimize

$$f(\mathbf{x}),$$

$$\mathbf{x} = \{x_1, x_2, .., x_n\}$$

Unconstrained optimization involves

- a vector of design variables

- an objective function.

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Example

Minimize

$$f(\mathbf{x}) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$

This is known as Rosenbrock's banana function, and it is really a difficult problem to solve numerically. Solving analytically is trivial.

If we only had explicit problems, we would have no problems!

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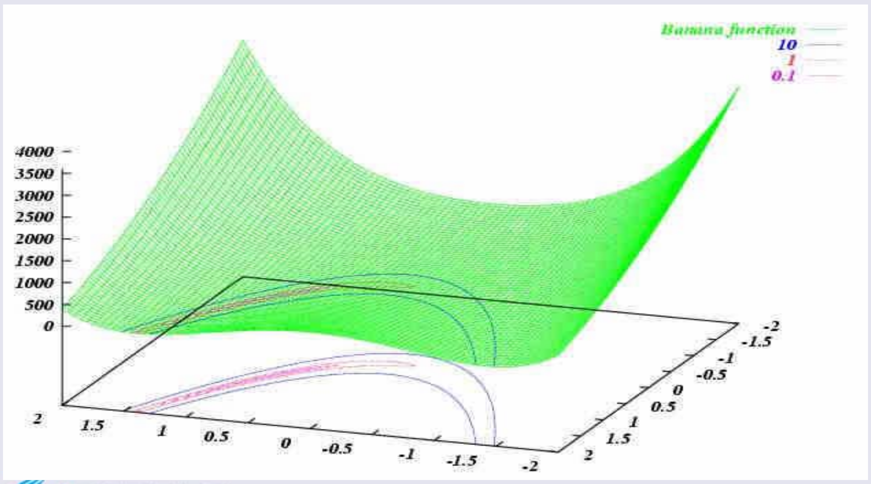
Rosenbrock's banana function

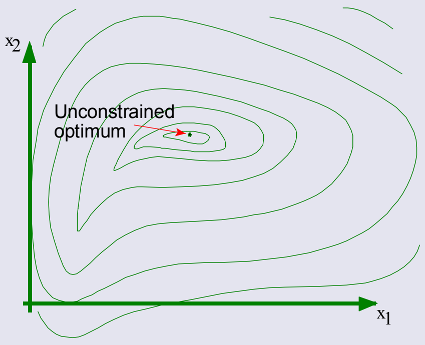
(http://www.fi.uib.no/~antonych/RGA4.html)



Unconstrained optimization

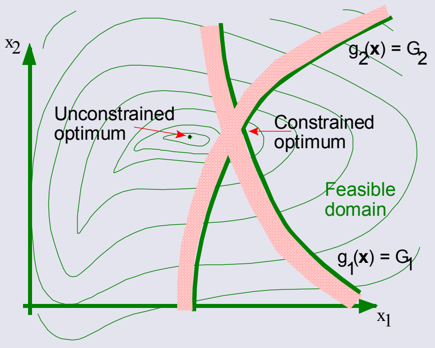
(Graphical interpretation)

- Unconstrained optima are easy to find analytically if you know the function you are working on explicitly.
- If you do not know the function, then you have to devise an algorithm.
- A minimization algorithm is like trying to find the lowest point in the landscape blindfolded.



Constrained Optimization

- Constrained optimization involves constraints that act like fences in the design domain.
- The constraints fence in the "feasible domain".
- The solution to the problem is the place inside the feasible domain that gives the smallest values of the objective function.
- There may also be equilibrium constraints present.



Mathematical formulation

Objective function

Design space

Minimize

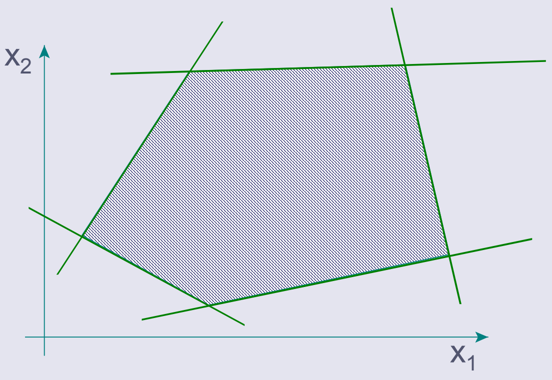
$$g_0(\mathbf{x}), \qquad \mathbf{x} = \{x_1, x_2, .., x_n\}$$

Subject to

$$g_i(\mathbf{x}) \le G_i \qquad i = 1..m$$

Constraints

Linear problems

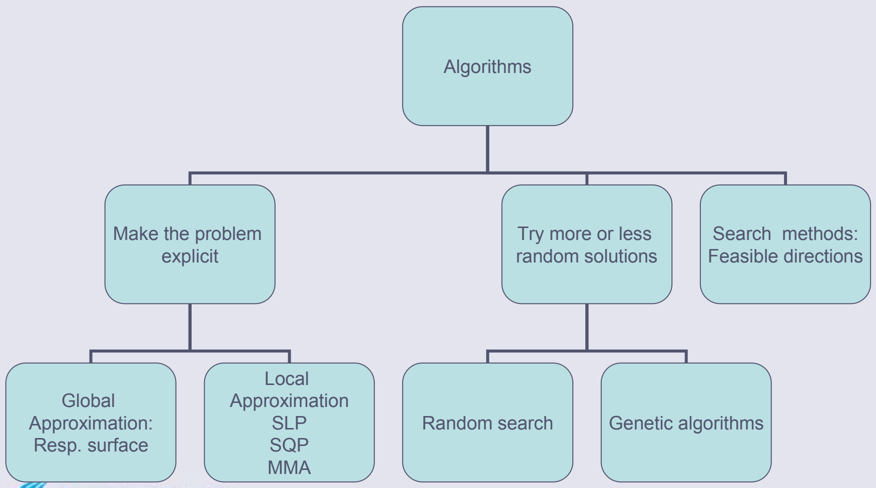


Linear programs are convex. They usually only have one optimum. If they have several optima, then they are all equally good.

Linear problems are much easier to solve than nonlinear problems.

Computers can solve linear problems with thousands of variables.

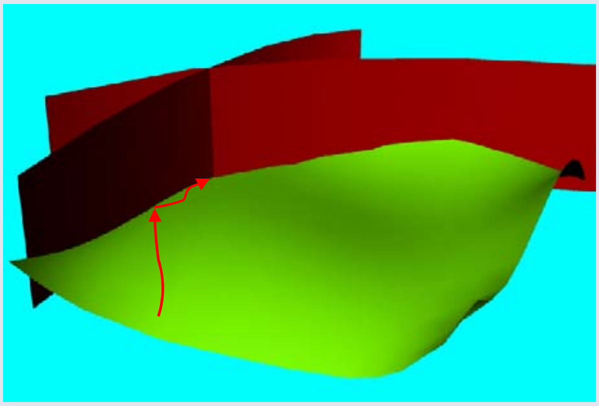
Algorithms


Algorithms
Make the problem explicit
Try more or less random solutions
Search methods: Feasible directions
Global Approximation: Resp. surface
Local Approximation SLP SQP MMA
Random search
Genetic algorithms

Formulation of optimization problems
- takes roughly half the time of the total effort to solve the problem

- Define a set of design variables. They form the variability of the problem – the design space.
- Formulate the constraints you may have.
- Define exactly one objective function.

Feasible directions



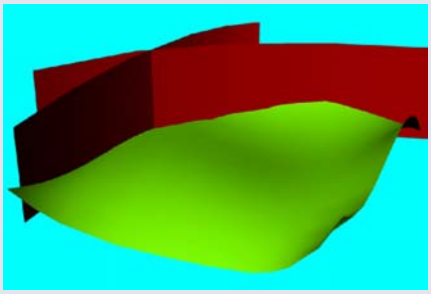
This is like walking blindfolded towards a low point.

We can only feel the slope of the ground under our feet.

We can feel the presence of the fences when we get close to them

Local approximations

- Create an approximation of the functions based on their values and gradients in the vicinity of our current point.
- Solve the approximative problem and hope it takes you in the right direction.
- Repeat the process until you reach the optimum.



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Local approximation of the problem

If we don't know the functions of the problem explicitly, we have to approximate it about the current point, $\mathbf{x}^{(k)}$.

Minimize

$$g_0(\mathbf{x}^{(\mathbf{k})}) + \nabla g_0(\mathbf{x}^{(\mathbf{k})})(\mathbf{x} - \mathbf{x}^{(\mathbf{k})}) + \ldots \qquad \mathbf{x} = \{x_1, x_2, x_3, \ldots, x_n\}$$

Subject to

$$g_i(\mathbf{x}^{(\mathbf{k})}) + \nabla g_i(\mathbf{x}^{(\mathbf{k})})(\mathbf{x} - \mathbf{x}^{(\mathbf{k})}) + \ldots \qquad i = \{1..m\}$$

The solution to the approximated problem will not be completely correct, but it is probably a step in the right direction.

Sensitivity analysis is finding the gradients of the functions of the problem, and this is an integral and very important part of optimization.

Sensitivity analysis is feeling which way the slope of the ground is under your feet.

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Sensitivity analysis

Local approximations as well as search methods require gradient information.

The simplest way to find the gradients is by forward finite difference:

$$\nabla g_i(\mathbf{x}^{(\mathbf{k})}) \approx \frac{g_i(\mathbf{x}^{(\mathbf{k})} + \Delta x_j) - g_i(\mathbf{x}^{(\mathbf{k})})}{\Delta x_j} \quad \text{for } j = 1..n$$

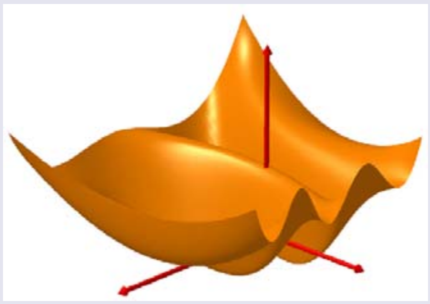
This requires one additional analysis for each variable and can be time consuming if we have many variables.

If you know more about the problem, then you might be able to do this much more efficiently!

Global approximations

- Start with a series of calculations of function values for different design variable combinations (design of experiments)
- Then fit global functions through the results.
- Solve an explicit optimization with the approximated functions.

Convexity



Ikke-konveks funktion

- Nice optimization problems are convex. They have only one optimum, and you can get to it from any place in the design space by walking down hill. Such problem can usually be solved.
- Non-convex problems are very difficult, and you can never be sure that you have found the global optimum.
- Non-convex problems are usually solved by more or less heuristic methods tailored to the problem.
- You rarely know in advance whether a problem is convex.

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What can go wrong?

- Unbounded problems: If the solution is that any of the variables should be ± ∞.
- Poles: If the objective function or constraints are undefined for certain values of the design variables.
- Non-smooth problems. The optimizer relies on sensitivity information. It will not work if the functions are not differentiable.
- No feasible solution. If no solution satisfies all the constraints, or if the initial guess violates some constraints.
- Local optima.

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How to select an algorithm

- Is the problem convex?
- Are the functions smooth?
- What is the size of the problem in terms of variabes and constraints?
- Can you obtain gradients and how difficult is it?
- Which practical methods do you have for hooking the optimizer up with the analysis?
- High-order algorithms require nice problems.

The efficiency of an algorithm is measured in terms of how many analyses are necessary before you find the optimum.