Hydrodynamic analysis and modelling of ships

Wave loading

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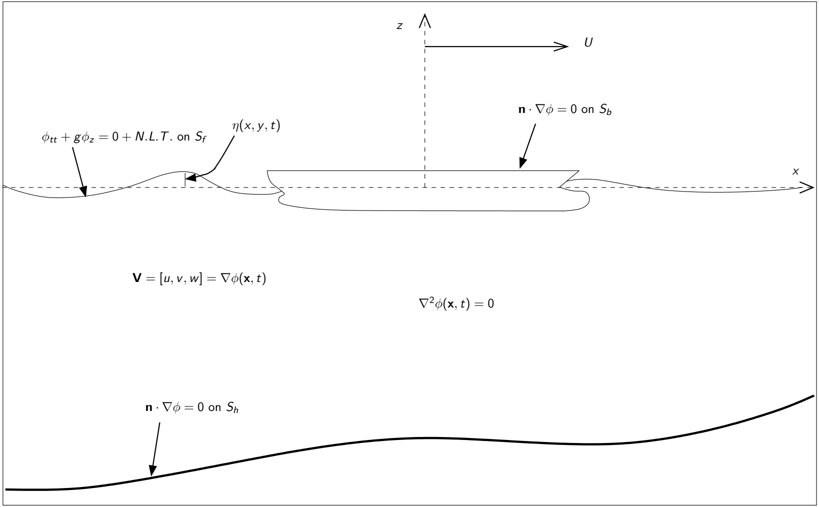
Wave loads on ships via potential flow theory

- Linear theory
  - Wave resistance
  - Motion response
  - Structural loading
- Second-order effects
  - Springing and Whipping
  - Added resistance, mean drift forces
- Higher-order effects
  - Parametric roll
  - Moored ship resonance response

A potential flow approximation (no viscosity/vorticity)



The first-order solution

Linearize the flow around a steady solution Φ^0
(Neumann-Kelvin: $\Phi^0 = -Ux$)

$$\Phi = \Phi^0(\mathbf{x}) + \epsilon\phi^1(\mathbf{x}, t) + \ldots, \qquad \epsilon \propto \frac{H}{L} \ll 1 \text{ (wave steepness)}$$

Newton's law - the equations of motion for the ship

$$\sum_{k=1}^{6} (M_{jk} + a_{jk})\,\ddot{x}_k(t) + \int_0^t K_{jk}(t-\tau)\,\dot{x}_k(\tau)d\tau + C_{jk}\,x_k(t) = F_{jD}(t)$$

$$j = 1, 2, \ldots, 6$$

$M_{jk},\ C_{jk}$: Body's inertia and hydrostatics

$K_{jk},\ a_{jk}$: Force due to radiation of waves by the body motions

F_{jD} : Force due to diffraction of the incident waves

Time-harmonic (frequency-domain) solution

Let $\eta(t) = \Re\{\mathcal{A}\,\mathrm{e}^{i\omega t}\}$, then $x_k(t) = \Re\{\tilde{x}_k(\omega)\,\mathrm{e}^{i\omega t}\}$ and

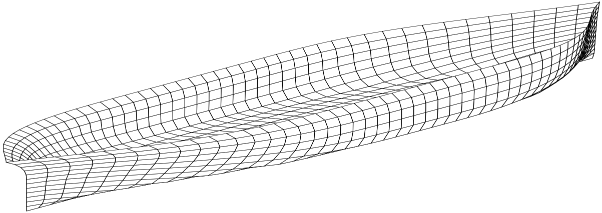
$$\sum_{k=1}^{6} \left\{-\omega^2\left[M_{jk} + A_{jk}(\omega)\right] + i\,\omega B_{jk}(\omega) + C_{jk}\right\} \tilde{x}_k(\omega) = \tilde{F}_{jD}(\omega)$$

$$j = 1, 2, \ldots, 6$$

where the IRF's and the FRF's are related by Fourier transform:

$$\begin{aligned} A_{jk}(\omega) &= \quad a_{jk} - \frac{1}{\omega}\int_0^\infty dt\ K_{jk}(t)\sin\omega t; \\ B_{jk}(\omega) &= \quad \int_0^\infty dt\ K_{jk}(t)\cos\omega t. \\ \tilde{F}_{jD}(\omega) &= \quad \int_0^\infty dt\ F_{jD}(t)e^{-i\omega t} \end{aligned}$$

Numerical solution - 3D panel methods



Green's theorem ($\phi_n \equiv \mathbf{n} \cdot \nabla\phi$):

$$0 = \int_{\mathcal{V}} (\nabla^2\phi\, G - \phi\, \nabla^2 G) dV = \oint_S (\phi_n\, G - \phi\, G_n) dS$$

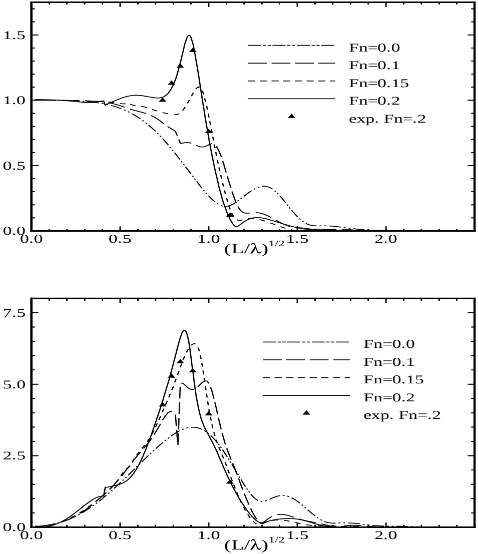
Where ϕ and G are two solutions to the Laplace equation.

- PDE in $\mathcal{V} \Rightarrow$ integral equation over the boundary S

$$\oint_S \phi\, G_n\, dS = \oint_S \phi_n\, G\, dS$$

An integral equation for ϕ on S.

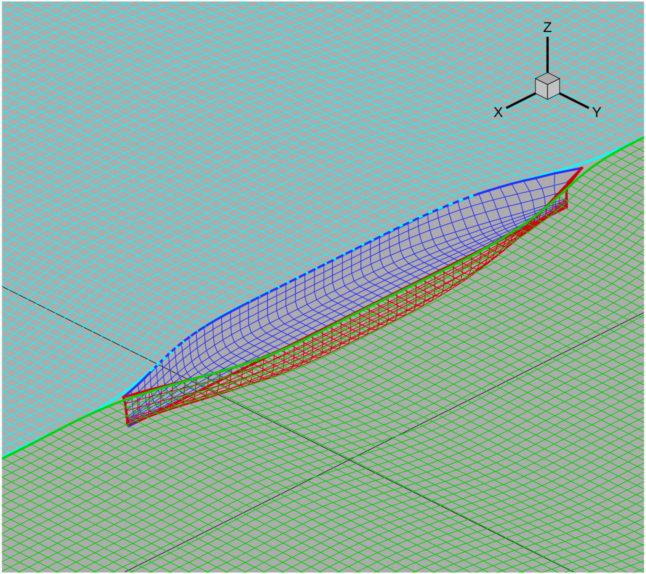
Motion response in waves (Free-Surface Green function) [1]



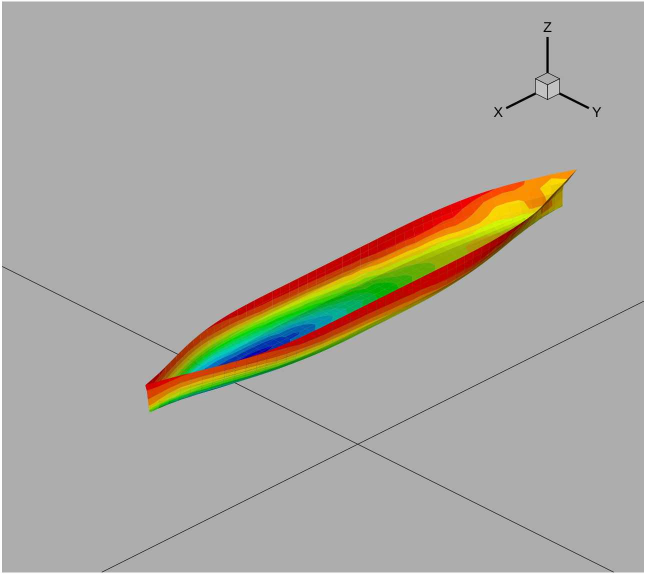
[1] Bingham et al, (1994). 20th Symp. Naval Hydrodynamics

Rankine-type panel methods, high-order panels

Free-surface and bottom boundaries are gridded ⇒ more flexibility

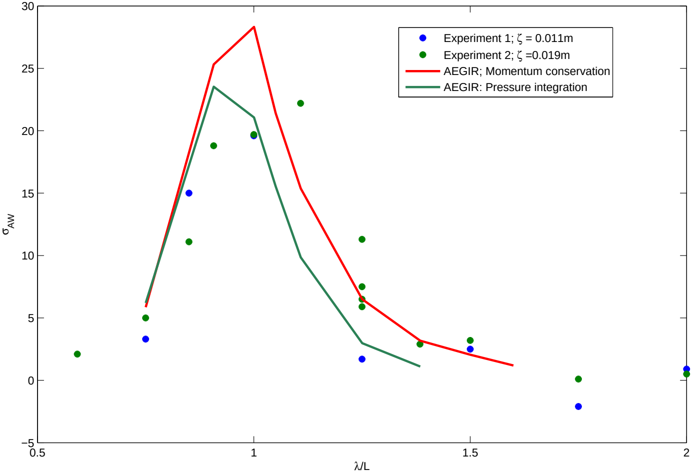


Pressure distribution on the hull



Added resistance in waves (2nd-order mean forces)[2]

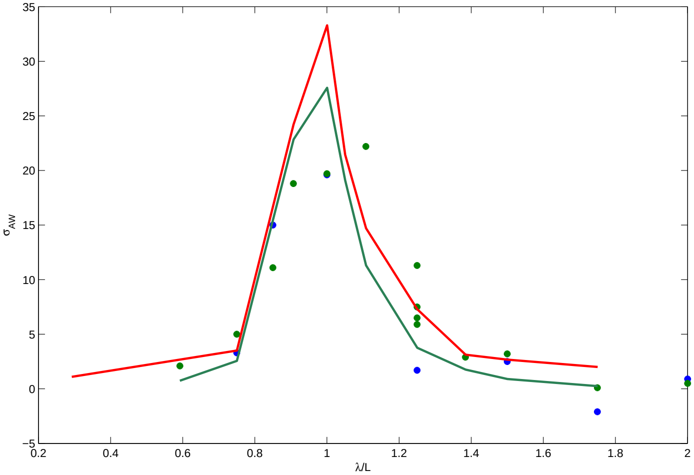
Neumann-Kelvin linearization



[2] Joncquez et al, (2008). 27th Symp. Naval Hydrodynamics

Added resistance in waves

Double-body linearization



Other solution methods - Strip theory

Two basic assumptions:

1. The body is thin: beam and draft << length. $ds \approx d\xi \, dl$

$$\int_{S_b} ds \rightarrow \int_L \int_{C_x} dl \, d\xi$$

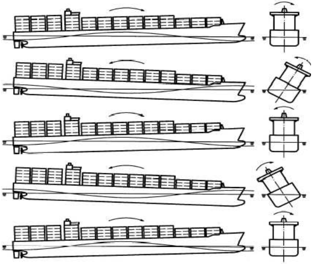
   3-D forces as integrals (sums) of sectional force coefficients.

2. Encounter frequency is high: $\omega >> U(\partial/\partial x)$, zero speed free-surface condition is assumed valid.

- Much faster than 3D methods.
- Good performance for linear motions but not for added-resistance.
- 2nd-order strip theory has been successful for springing. [3]

[3] Vidic-Perunovic, J, and Jensen, JJ, (2005). Marine Structures.

Parametric Roll



Simple model with linear surge and heave motions and dynamic calculation of the roll GM, can predict the occurrence well. [4]

[4] Vidic-Perunovic, and Jensen (2009). Submitted to *Ocean Engineering*.

A hybrid model for nonlinear moored ship motions[5]

Assumptions

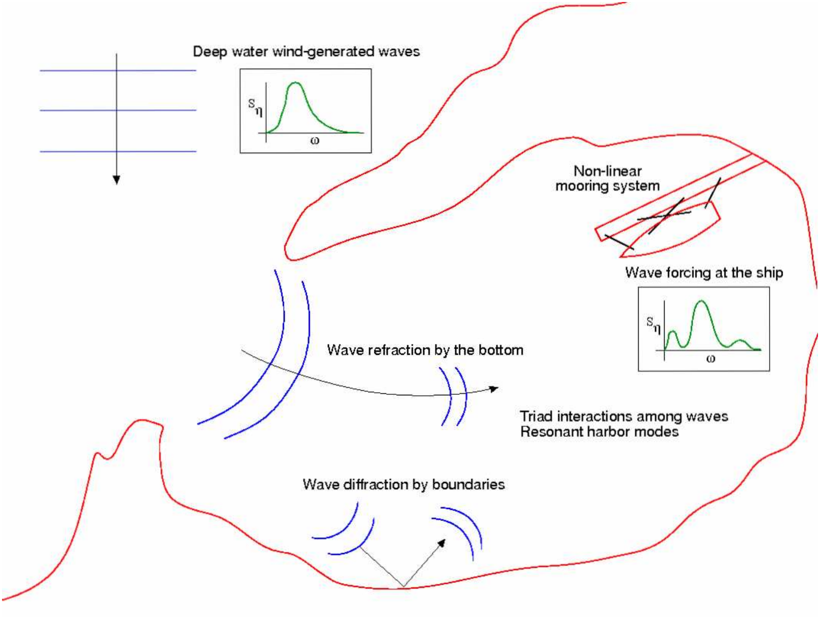
- Small wave steepness at the ship (linear hydrodynamics)
- Mooring system adds: Nonlinearity, and resonant horizontal modes
- Body motions are still small

Harbor and ship problems are de-coupled

1. Wave forcing: Boussinesq-type model (MIKE 21BW)
2. Wave-body interaction: Panel method
3. Ship response: Time-domain equations of motion with nonlinear external forcing

[5] Bingham (2000). *Coastal Engineering*

Pictorial representation of the situation



Add a nonlinear forcing term to the equations of motion

$$\sum_{k=1}^{6} (M_{jk} + a_{jk})\, \ddot{x}_k(t) + \int_0^t K_{jk}(t-\tau)\, \dot{x}_k(\tau) d\tau + C_{jk}\, x_k(t) =$$

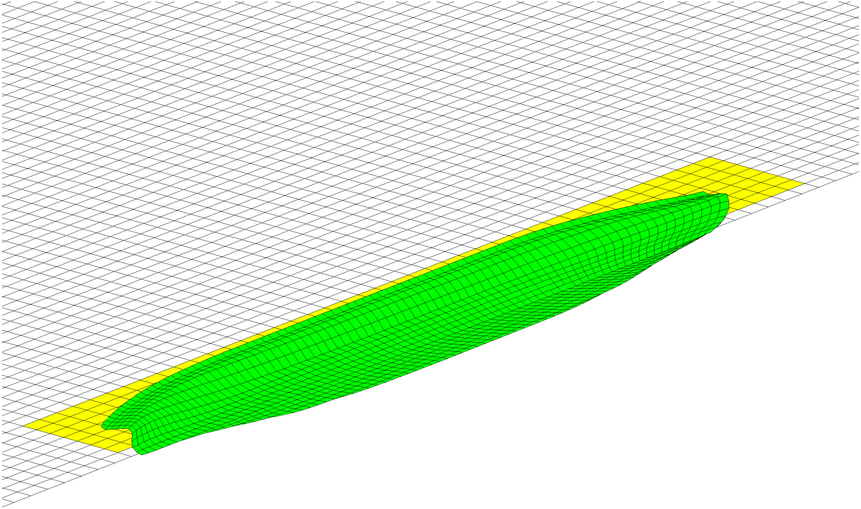
$$F_{jD}(t) + \underline{F_{j\mathrm{nl}}(t)};$$

$$j = 1, 2, ..., 6$$

$F_{j\mathrm{nl}}$: Nonlinear forcing. Mooring lines, fender friction, viscous damping, ...

Wave forcing $F_{jD}(t)$ via linearization and interpolation from the Boussinesq grid to the panel method grid

Bi-linear interpolation and linearization.



Linearization of the Boussinesq solution

Boussinesq solution $\rightarrow \eta,\ [u', v'] = 1/(h+\eta) \int_{-h}^{\eta} [u, v]\, dz$

$$p_0(\mathbf{x}, t) = p(\mathbf{x}, 0, t) = \rho g \eta(\mathbf{x}, t)$$
$$w_0(\mathbf{x}, t) = w(\mathbf{x}, 0, t) = \dot{\eta}(\mathbf{x}, t) \qquad \text{linear FSBC}$$

Fourier transform to get $\tilde{p}_0(\mathbf{x}, \omega),\ \tilde{u}'(\mathbf{x}, \omega),\ \tilde{v}'(\mathbf{x}, \omega),\ \tilde{w}_0(\mathbf{x}, \omega)$.

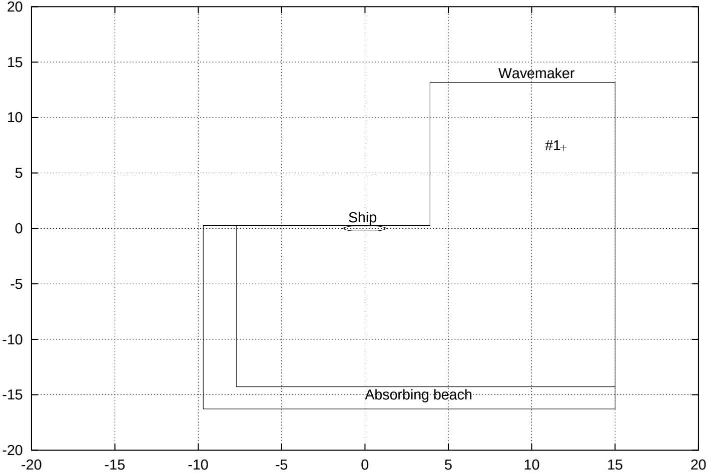
$$\tilde{c}(\mathbf{x}, z, \omega) = \left\{ \tilde{c}_0(\mathbf{x}, \omega) \frac{\cosh[k(z+h)]}{\cosh(kh)} \right\}, \qquad c = u, v, p;$$
$$\tilde{w}(\mathbf{x}, z, \omega) = \left\{ \tilde{w}_0(\mathbf{x}, \omega) \frac{\sinh[k(z+h)]}{\sinh(kh)} \right\}$$

$$\omega^2 = gk \tanh(kh)$$

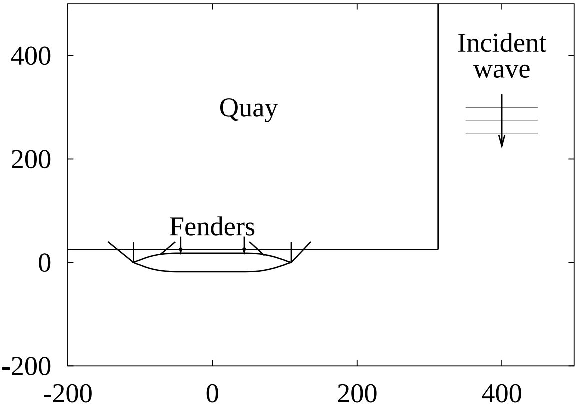
$$(\tilde{u}_0, \tilde{v}_0) = (\tilde{u}', \tilde{v}') \frac{kh}{\tanh(kh)}$$

A nonlinear test case:

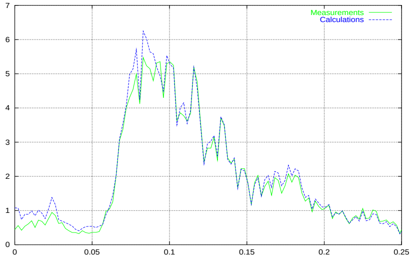
A ship moored in an L-shaped harbor



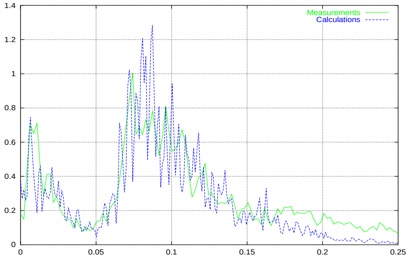
Plan view of the experimental setup, lengths are in meters. The model- to full-scale factor is 80, and the water depth is 0.3 meters.



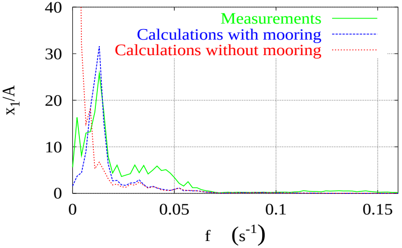
The mooring arrangement with respect to the ship's waterline. The ship is an LPG tanker $C_b = .816$. Lines and fenders are linear springs. (Shown at full scale.)



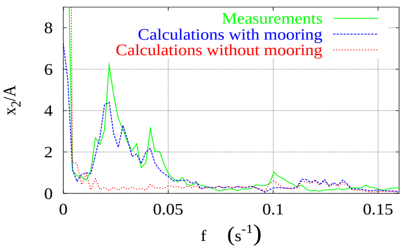
η_I at w.g. #1



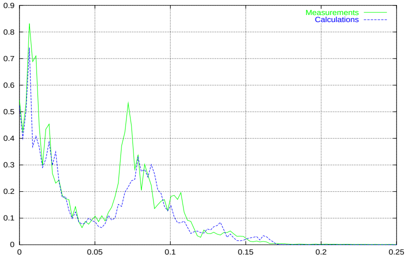
η_I at the ship



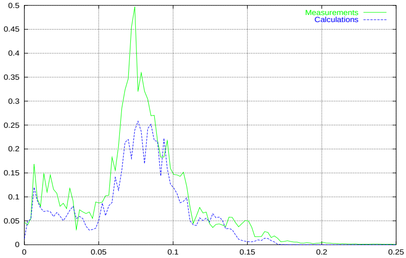
Surge



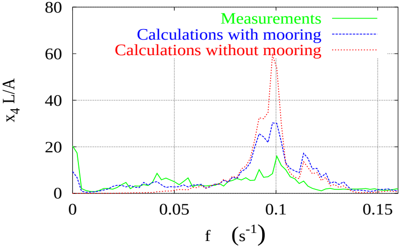
Sway



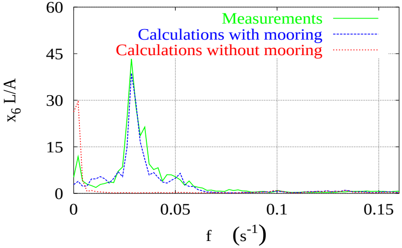
Heave



Pitch



Roll



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Conclusions

- Potential flow solutions to first- and second-order provide a good overall analysis of the most important wave induced loading factors for ships
- Hybrid models can efficiently include more physics in a mostly potential flow solution (mooring systems, empirical viscous damping forces, etc.)